Consistency of nuclear mass formulae

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Abstract. – The general scepticism and loss of faith on the predictive ability of different mass formulae, arising out of the divergence of their predictions in unknown regions taken with respect to a reference mass formula, is successfully dispelled. When the result of relativistic mean field (RMF) theory with a Lagrangian common for all nuclei is taken as reference, the divergence disappears, and clear trend with strong correlation appears restoring our faith in general on the predictions of mass formulae, qualifying them as useful guideline for theoretical and experimental studies of nuclear phenomena.

Mass formulae occupy the centre stage in the research in nuclear physics. The first model of the nucleus is a mass formula proposed by Bethe and Weiszackar in mid 1930s, soon after the discovery of the neutron unravelling the composition of the nucleus. It defined some of the key concepts and parameters for the first time for the description of nuclear phenomena, and laid the foundation for their future exploration. Our inability to predict the masses of nuclei starting from first principle, and our present experimental incapability to produce large majority of them in the laboratory, warrants reliable mass formulae of nuclei for understanding many phenomena, most notably, the nucleosynthesis and stellar evolution. Therefore the development of nuclear mass formulae has been all along a core theme of nuclear physics which has implicitly sustained and nourished the research in diverse areas of nuclear structure and nuclear reactions. Over the last 70 years, about a dozen of mass formulae have been proposed.

With the advent of heavy-ion reactions, the prospect of the exploration of the "terra incognita" is very much in the realm of possibility. To the already known about 2000 nuclei, another 5000 to 7000 nuclei will be added in future by their synthesis in the laboratory for which unprecedented activity in different laboratories is underway. Therefore the predictive ability of differnt mass formulae is under serious scrutiny. Figure 1 represents an often quoted result [1–3] on the comparison of the predictions of different mass models with experiment on the Tin isotopes. Here the difference of the binding energy of various models and experimental results with respect to the predictions of the model of Duflo and Zuker [4], taken as reference, is plotted for the chain of isotopes of Sn with neutron number varying between 45 and 110. There is unanimity of all the mass models on good agreement with experiment in the known region close to stability, however, the predictions diverge as one moves away to unknown

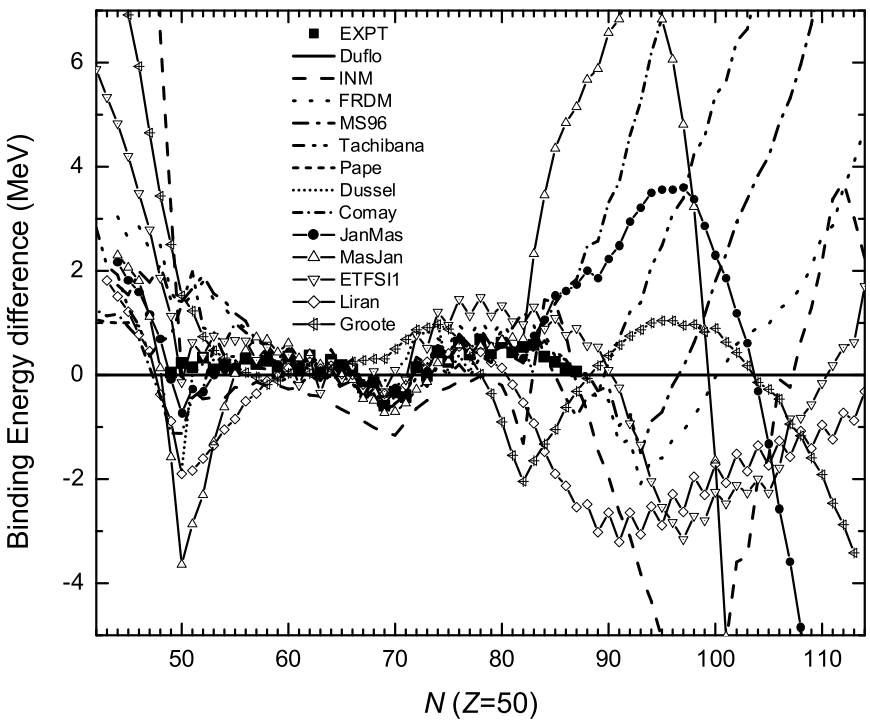

Fig. 1 – Binding energy difference in Sn isotopes with the mass formula of Duflo and Zuker [4] as reference. The data corresponding to Pape, Dussel, MS96, FRDM, Comay, Tachibana, JanMas, MasJan are taken from the tables given in Ref. [5]; Liran and Groote are extracted from Fig. 1 of Ref. [2]; ETFSI1 from Ref. [6].

regions on either sides where measurements have not been feasible. To see if the same feature persists in other regions, we have plotted corresponding diagrams in Fig. 2 for the isotopes of $Z = 8$, 20, 82, and the results are similar to Fig. 1. Since the masses of the known regions have been used in the fit by all the mass models, the agreement with data in these regions is to be expected only; what is worrying is in the unknown regions they do not exhibit a common trend (like rising or falling), but diverge without any correlation. This divergence with the same intensity is seen when the mass model of Möller et al. [7] is used as reference as seen in Fig. 4 of Ref. [1]. We have examined the results of microscopic methods [6], to see if they can be used as reference yielding common trend for the predictions of mass formulae. The results are only similar to Fig. 1. It is this divergence which has raised question about the efficacy of the mass models, and more importantly shaken the faith of the nuclear physics community on the predictive ability of the mass models as a whole. One is constrained to think whether development of different mass models is a parameter game only without worthwhile theoretical foundation. In this letter, we address this issue and show that the above contention is incorrect, and the mass formulae are in general consistent, and should be relied upon as guideline.

The basic question that perturbs us is, why the predictions of different mass formulae taken with respect to that of Duflo-Zuker mass formula as reference, shows the divergence in the unknown region ? It is expected that the prediction of each mass formula in the unknown region will not agree 100% with the future data (when measured) and the discrepancy will

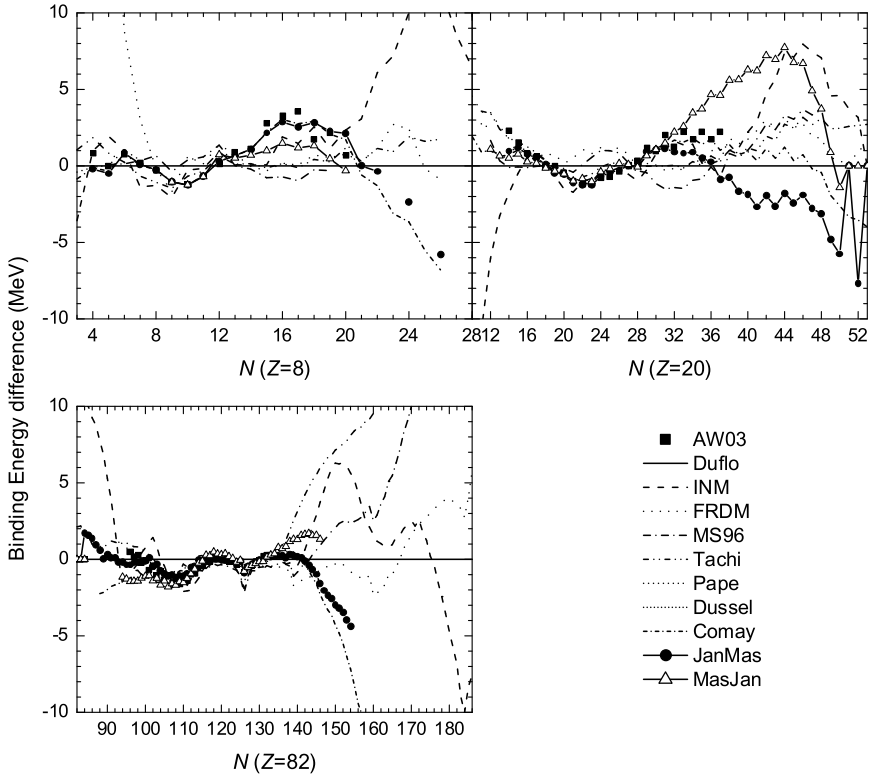

Fig. 2 – Binding energy difference in O, Ca, Sn and Pb isotopes with the mass formula of Duflo and Zuker [4] as reference.

go on increasing as one moves farther away from the known region. Further, it is a common feature with most mass formulae, that the degree of success varies from region to region even in known domains, and quite likely, to be more so in unknown domains. This is due to inadequate accounting of the variation of the characteristic local structure effects originating from shell, deformation etc. The mass formulae of Duflo-Zuker and Möller et al are not free from this discomfiture. It is quite likely, the degree of accuracy of the predictions of a test mass formula region-wise, may not match with those of the reference mass formula, leading to randomness when the differences are calculated. Therefore predictions of different mass formulae taken with respect to reference mass formula may show randomness manifesting no common trends which appear as divergence in Fig. 1. It is like making measurements in a unsteady co-ordinate system. A fixed common substratum as reference is needed with respect to which the predictions of different mass formulae have to be obtained and their variations be studied. If they do not show any common trend, and exhibit divergence in the unknown regions, then one should not repose any faith in their predictions.

It must be recognised that each mass formula, in its own way, tries to simulate the effect of the nuclear many-body Hamiltonian in a semi phenomenological manner using parameters. All mass models based on liquid drop picture assume that the energy of a classical liquid drop added to the shell correction energy — using a mean field supposed to arise from the nuclear Hamiltonian — would represent the true ground-state energy of a nucleus. The Comay-Kelson [8] mass model rests on the goodness of nuclear mean field picture and the Hartree-

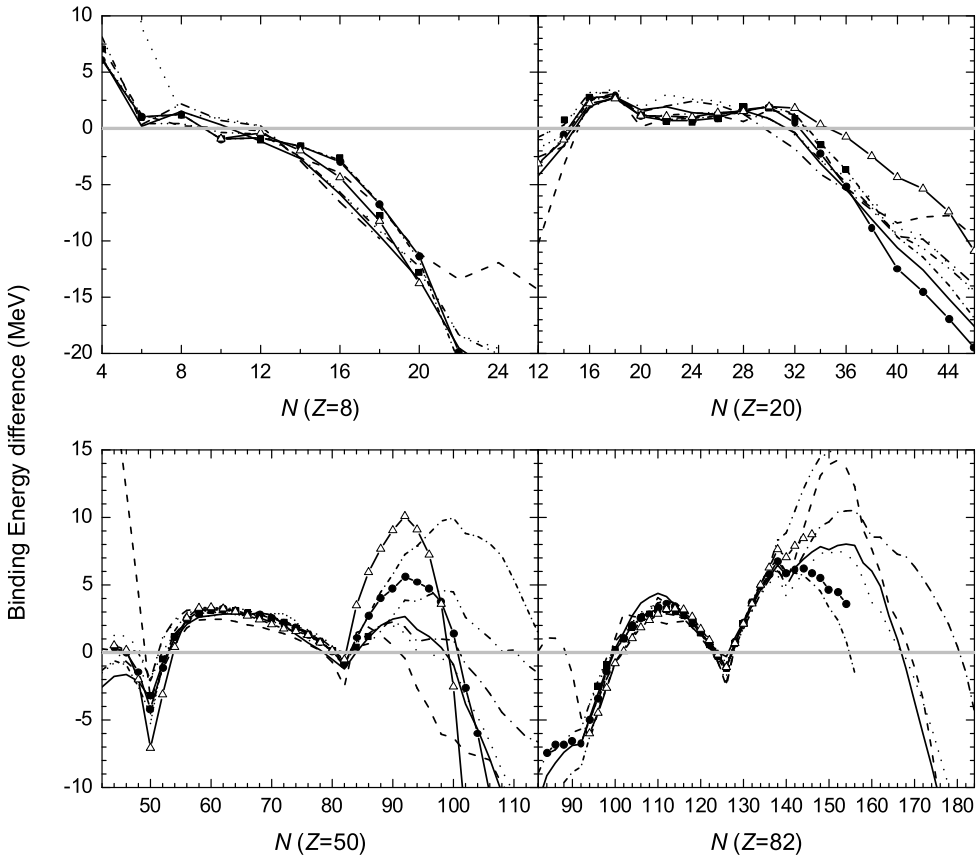

Fig. 3 – Binding energy difference in O, Ca, Sn and Pb isotopes with the results of RMF calculations [9] [with NL3 parameter set [10]] as reference.

Fock (HF) description of the ground state of the nucleus. The Duflo-Zuker mass model goes beyond HF assuming the monopole part of the interaction determines the mean field and the multipole terms act as the residual interaction giving rise to configuration mixing. Thus every mass formula has a foundation of its own rooted in the nuclear many-body Hamiltonian, and therefore expected to show, in general a common trend, although quantitatively they may differ from one another in their predictions, depending upon the efficacy of their simulation of nuclear dynamics. With this surmise, we believe, if a common substratum with strong theoretical validity for all regions is found in the form of reference mass model, then the predictions with respect to it in the unknown region will quite likely show a common trend, but no divergence.

The binding energy calculated in RMF theory with an appropriate field theoretic Lagrangian, can serve as a common substratum. With a microscopic Lagrangian with the parameter set like NL3 [10] supposed to be valid for all nuclei, and the well defined RMF formalism being applied without any variation from one region to another, the binding energies so obtained can qualify to be a good reference. In Fig. 3, the results of such calculations for 10 mass formulae are presented for the isotopes of $Z = 8$, 20, 50 and 82 along with the experimental data. It can be seen that for each chain of isotopes for the O, Ca, Sn and Pb

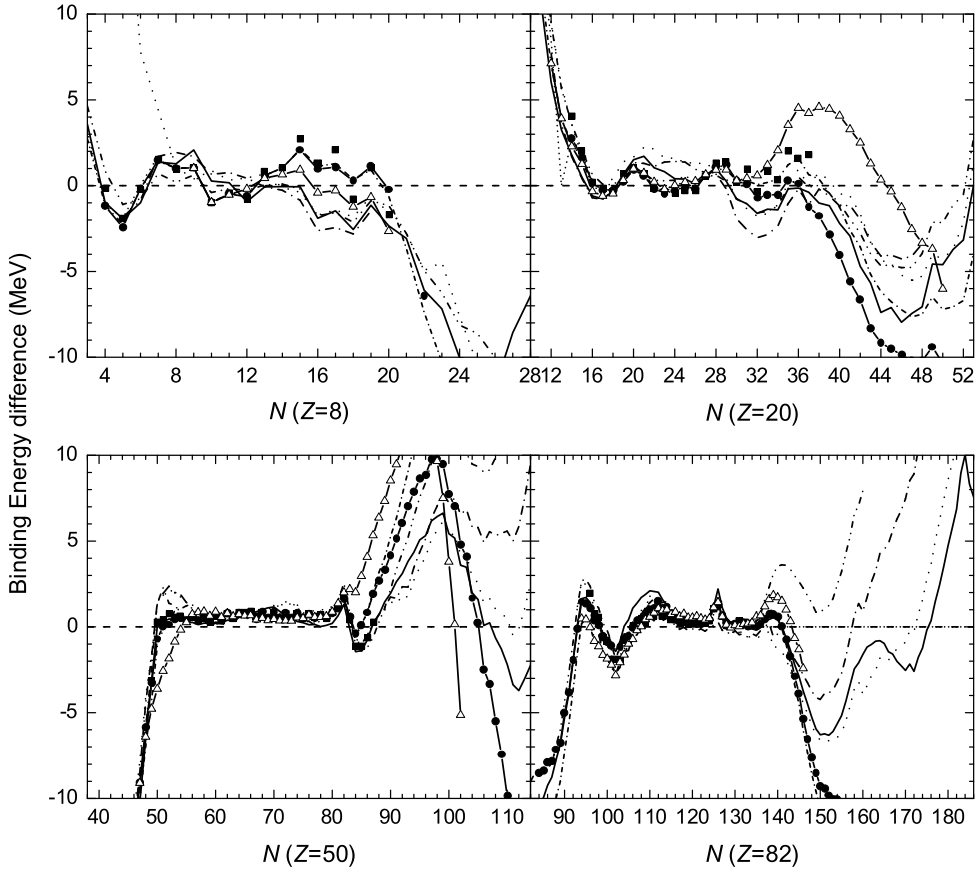

Fig. 4 – Binding energy difference in O, Ca, Sn and Pb isotopes with the results of INM mass formula [12] as reference.

elements, the predictions in the unknown regions, show in general a common trend for all the 10 mass formulae although quantitatively they differ from one another. The divergence seen in Fig. 1 and Fig. 2 has disappeared in Fig. 3 in conformity with our expectation. Similar results are also obtained with NL-SH, G1 and G2 parameters [11]. Therefore the predictions of the mass formulae in the unknown regions are not arbitrary or random supposed so far, rather show definite trend which can be used as useful guideline for theoretical and experimental studies.

The infinite nuclear matter (INM) mass model [12] is based on many-body theoretic foundation. It is built in terms of quantum mechanical nuclear liquid rather than the classical liquid used in Bethe-Weiszacker-like mass models. The quantum effect is taken into account in this model by the use of generalized Hugenholtz-Van Hove (HVH) theorem [13] which also accounts the effect of three-body force [14]. In view of its unique success like prediction of saturation properties and incompressibility [14,15] of INM from nuclear masses, shell quenching [16] in large neutron shells N=82, 126 and broadening of stability peninsula [17], we use it as the reference mass formula to see its general validity. The results so obtained for the 10 mass formulae are presented in Fig. 4. It is pleasing to find that no divergences appear in unknown regions, but common trends — as found in Fig. 3 pertaining to the case with mi-

croscopic RMF calculations as reference — are seen. This is indicative of the general validity and uniqueness of INM mass formula.

In summary, the notion propagated in the last several years that the predications of different mass formulae do not show any correlation and diverge as one moves to the unknown region is not correct. This happens because of uneven degree of inaccuracy in various domains in the predictions of reference mass formula with respect to which the results of other mass formulae are calculated. When predictions of the RMF theory with a fixed Lagrangian of general validity for all nuclei is taken, the divergence disappears and common trends for all mass models manifest. The INM mass model as reference also reproduces the common trend. The general scepticism about the predictability of mass formulae is then dispelled and our faith on the soundness of their results is restored. Their predictions can be used as useful guide in the theoretical and experimental exploration of the nuclear phenomena.

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